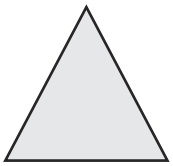
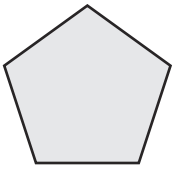
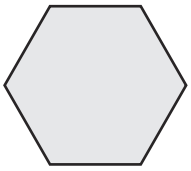
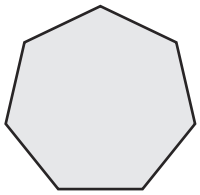
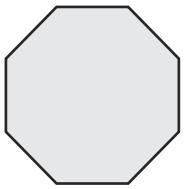
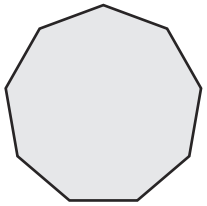
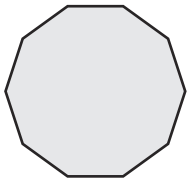
Angles in Polygons **Answers**

1. Complete the table. The first one has been completed for you.
   Note: each polygon is regular.

Polygon	Sum of Interior Angles	Interior Angle	Exterior Angle
 **Triangle**	$(n - 2) \times 180°$ $(3 - 2) \times 180$ 180°	180 ÷ 3 60°	360 ÷ 3 120°
 **Rectangle**	**$(n - 2) \times 180°$** **$(4 - 2) \times 180$** **360°**	**360 ÷ 4 or 180 – 90** **90°**	**360 ÷ 4** **90°**
 **Pentagon**	**$(n - 2) \times 180°$** **$(5 - 2) \times 180$** **540°**	**540 ÷ 5 or 180 – 72** **108°**	**360 ÷ 5** **72°**
 **Hexagon**	**$(n - 2) \times 180°$** **$(6 - 2) \times 180$** **720°**	**720 ÷ 6 or 180 – 60** **120°**	**360 ÷ 6** **60°**
 **Heptagon**	**$(n - 2) \times 180°$** **$(7 - 2) \times 180$** **900°**	**900 ÷ 7 or 180 – 51.4** **$128.\dot{5}71428\dot{}$** **128.6°**	**360 ÷ 7** **$51.\dot{4}28571\dot{}$** **51.4°**
 **Octagon**	**$(n - 2) \times 180°$** **$(8 - 2) \times 180$** **1080°**	**1080 ÷ 8 or 180 – 45** **135°**	**360 ÷ 8** **45°**
 **Nonagon**	**$(n - 2) \times 180°$** **$(9 - 2) \times 180$** **1260°**	**1260 ÷ 9 or 180 – 40** **140°**	**360 ÷ 9** **40°**

 **Decagon**	**$(n - 2) \times 180°$** **$(10 - 2) \times 180$** **1440°**	**1440 ÷ 10 or 180 – 36** **144°**	**360 ÷ 10** **36°**

2. Work out the sum of the interior angles for a polygon with:

   a. 20 sides

      (20 – 2) × 180

      3240°

   b. 45 sides

      (45 – 2) × 180

      7740°

   c. 100 sides

      (100 – 2) × 180

      17 640°

3. The interior angles of a polygon add up to 2880°. Work out the number of sides the polygon has.

   2880 ÷ 180 = 16

   16 + 2 = 18 sides

4. The interior angles of a polygon add up to 1980°. Work out the number of sides the polygon has.

   1980 ÷ 180 = 11

   11 + 2 = 13 sides

5. The interior angles of a polygon add up to 3060°. Work out the number of sides the polygon has.

   3060 ÷ 180 = 17

   17 + 2 = 19 sides

6. Calculate the size of each exterior angle in a regular polygon which has:

   a. 6 sides

      360 ÷ 6 = 60°

   b. 10 sides

      360 ÷ 10 = 36°

   c. 15 sides

      360 ÷ 15 = 24°

   d. 20 sides

      360 ÷ 20 = 18°

   e. 50 sides

      360 ÷ 50 = 7.2°

7. A regular polygon has an exterior angle of 36°. Calculate the number of sides to the regular polygon.

   360 ÷ 36 = 10 sides

8. A regular polygon has an interior angle of 175°. Calculate the number of sides to the regular polygon.

   180 – 175 = 5°

   360 ÷ 5 = 72 sides

Challenge

A regular polygon has an interior angle that is five times larger than its exterior angle. How many sides does the regular polygon have?

$x + 5x = 180$

$6x = 180$

$x = 30$

360 ÷ 30 = 12 sides

Angles in Polygons

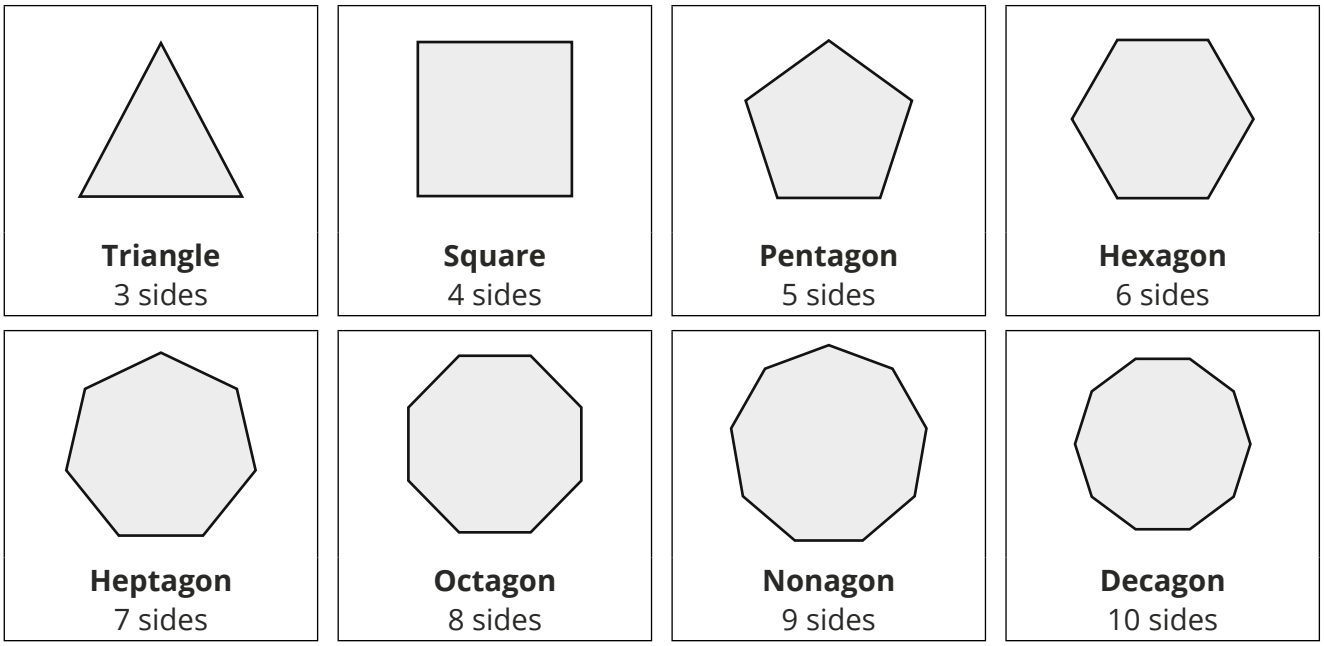
Prior Knowledge:

- Measuring angles.
- Substituting numbers into formulae.
- Know that angles in triangles sum to 180°.
- Know that angles on a straight line sum to 180°.

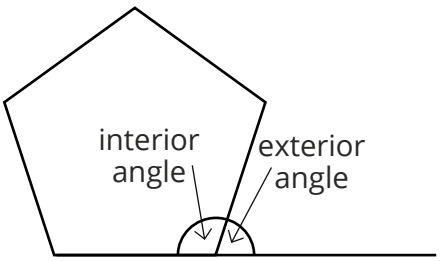
A **polygon** is a 2D shape which has 3 or more straight sides.

In a **regular** polygon, all the sides are the same length and all the **angles** are equal.

You need to know the names of these **regular** polygons and how many sides they have.



You also need to know how to find the **interior** and **exterior** angles of regular polygons.



An **interior** angle and its corresponding **exterior** angle add up to 180°.

Exterior Angles

The sum of the exterior angles of a polygon is always 360°. In a regular polygon, to find an exterior angle, you can divide 360° by the number of sides:

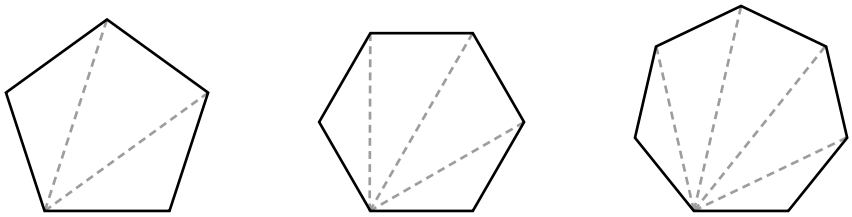
Exterior angle = $\frac{360}{n}$

Interior Angles

The formula for the sum of the interior angles in a polygon, with n sides, is:

Interior angle sum = $(n - 2) \times 180°$

This formula works for regular and irregular polygons. You can show how it works by dividing a polygon into triangles:

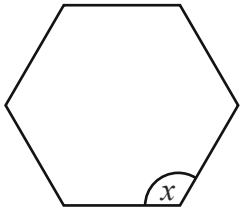


You can divide a pentagon into 3 triangles, a hexagon into 4 triangles and a heptagon into 5 triangles. Notice that, for each shape, you can find the number of triangles by taking two from the number of sides. This gives us $(n - 2)$.

The angles in a triangle always sum to 180°. This means you can multiply the number of triangles, $(n - 2)$, by 180 to get the total of the interior angles of the shape.

Because the angles in a regular shape are all equal, you can divide the sum of the interior angles by the number of sides to find the size of each interior angle.

Example 1: A regular hexagon is shown below. Calculate the size of the angle marked x.



Use the formula to find the sum of the interior angles.

$(n - 2) \times 180 = (6 - 2) \times 180$

$= 720°$

As the hexagon is regular, all the interior angles are equal. Therefore, to find the size of the interior angle, divide the sum of the interior angles by the number of sides, 6:

$x = 720 \div 6$

$= 120°$

Alternatively, the interior angle can be found by subtracting the exterior angle from 180°:

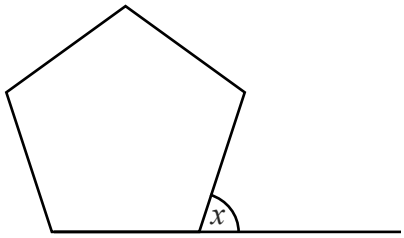
exterior angle = $720 \div 6$

$= 120°$

$x = 180 - 60$

$= 120°$

Example 2: A regular pentagon is shown below. Calculate the missing angle marked x.



The sum of the exterior angles for a polygon is 360°.

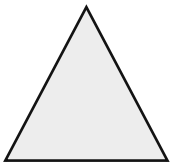
As the pentagon is regular, all the exterior angles are equal. Therefore, to find the missing angle, divide the sum of the exterior angles by the number of sides, 5:

$x = 360 \div 5$

$= 72°$

Your turn

1. Complete the table. The first one has been completed for you.
   Note: each polygon is regular.

Polygon	Sum of Interior Angles	Interior Angle	Exterior Angle
 **Triangle**	$(n - 2) \times 180°$ $(3 - 2) \times 180$ $180°$	$180 \div 3$ $60°$	$360 \div 3$ $120°$
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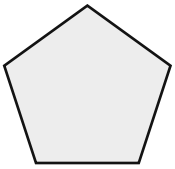
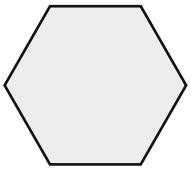
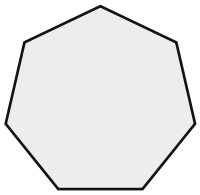
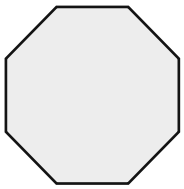
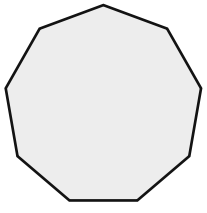
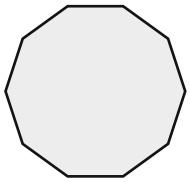
 **Pentagon**			
 **Hexagon**			
 **Heptagon**			
 **Octagon**			
 **Nonagon**			

 **Decagon**			

1. Work out the sum of the interior angles for a polygon with:

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2. The interior angles of a polygon add up to 2880°. Work out the number of sides the polygon has.

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1. Calculate the size of each exterior angle in a regular polygon which has:

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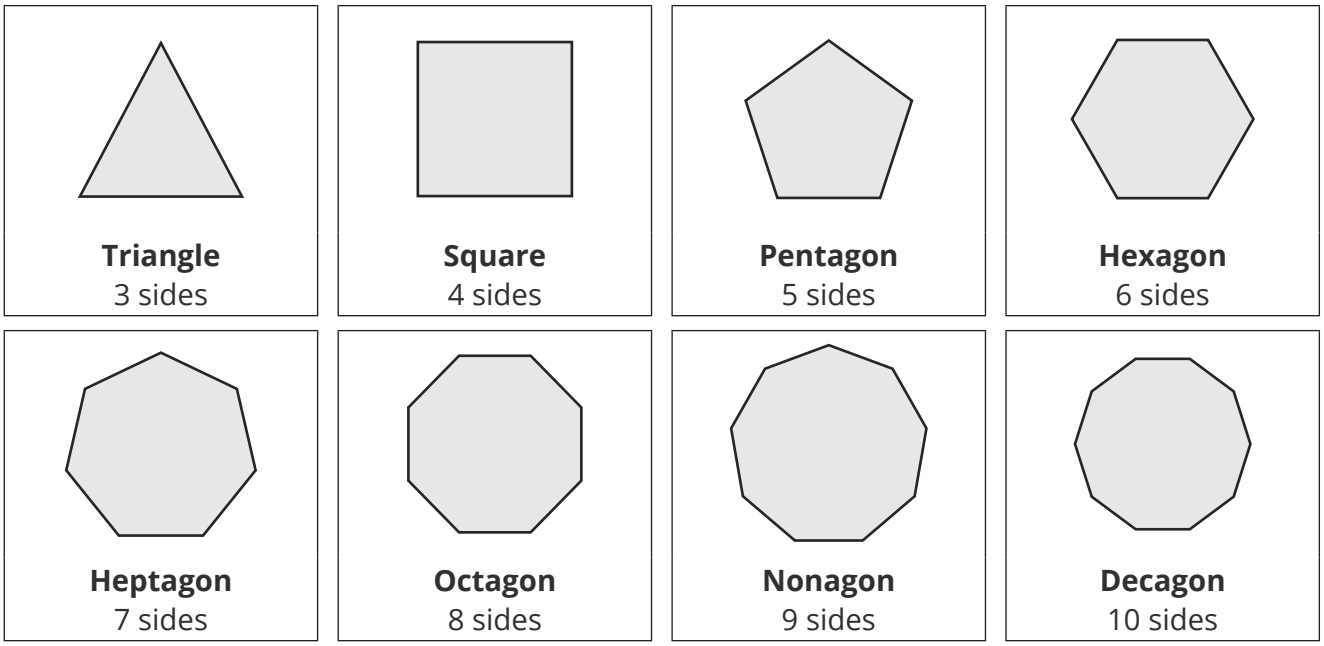

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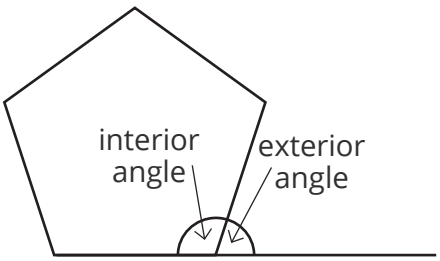

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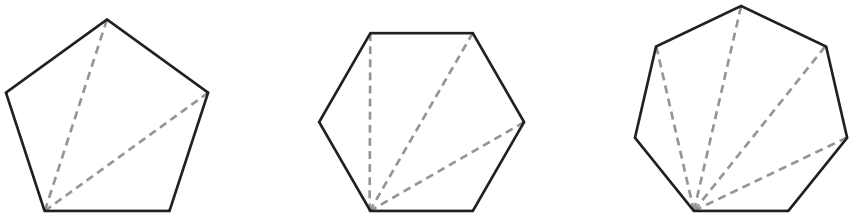

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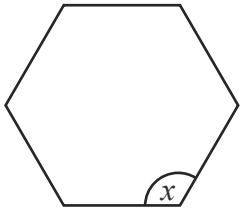

Use the formula to find the sum of the interior angles.

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$= 720°$

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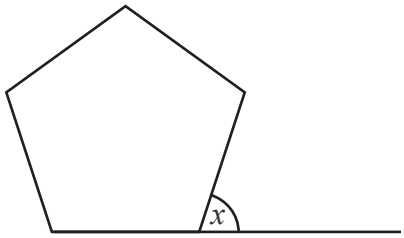

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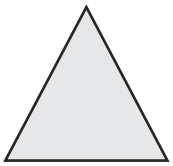 **Triangle**	$(n - 2) \times 180°$ $(3 - 2) \times 180$ $180°$	$180 \div 3$ $60°$	$360 \div 3$ $120°$
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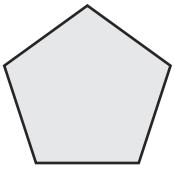
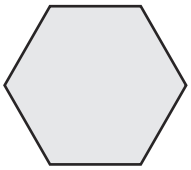
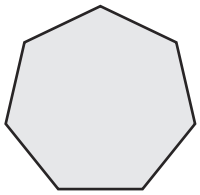
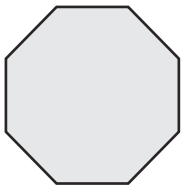
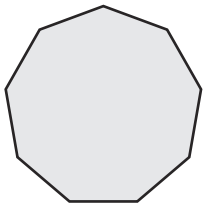
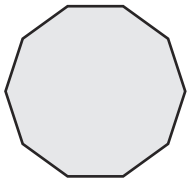
 **Pentagon**			
 **Hexagon**			
 **Heptagon**			
 **Octagon**			
 **Nonagon**			

 **Decagon**			

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